quantities and prices. In particular, budget shares and elasticities do not depend on price levels, but only on relative prices. Consequently it can be much easier to apply Walras' Law when it is written as (2.3) than when it is written as (2.2).

2.5.8 Walras' Law and Changes in Wealth: Elasticity Form

Not to belabor the point, but we can also write (2.1) in terms of elasticities, this time using the wealth elasticity, $\varepsilon_{iw} = \frac{\partial x_i}{\partial w} \cdot \frac{w}{x_i}$. Multiplying (2.1) by $\frac{x_i w}{x_i w}$ yields:

$$\begin{aligned} \sum_i \frac{p_i x_i}{w} \frac{w}{x_i} \frac{\partial x_i(p,w)}{\partial w} &\equiv 1 \qquad (2.4) \\ \sum_i b_i(p,w)\, \varepsilon_{iw} &= 1. \end{aligned}$$

The wealth elasticity ε_{iw} gives the percentage change in consumption of good i induced by a 1% increase in wealth. Thus, in response to an increase in wealth, total spending changes by ε_{iw} weighted by the budget share $b_i(p,w)$ and summed over all goods. In other words, if wealth increases by 1, total expenditure must also increase by 1. Thus, equation (2.4) is yet another statement of the fact that the consumer always spends all of her money.

2.6 Requirement 2 Revisited: Demand is Homogeneous of Degree Zero.

The second requirement for consumer choices is that "only real opportunities matter." In mathematical terms this means that "demand is homogeneous of degree zero," or:

$$x(\alpha p, \alpha w) \equiv x(p,w)$$

Note that this is an identity. Thus it holds for any values of p and w. In words what it says is that if the consumer chooses bundle $x(p,w)$ when prices are p and income is w, and you multiply all prices and income by a factor, $\alpha > 0$, the consumer will choose the same bundle after the multiplication as before, $x(\alpha p, \alpha w) = x(p,w)$. The reason for this is straightforward. If you multiply all prices and income by the same factor, the budget set is unchanged. $B_{p,w} = \{x : p \cdot x \leq w\} = \{x : \alpha p \cdot x \leq \alpha w\} = B_{\alpha p, \alpha w}$. And, since the set of bundles that the consumer could choose is not changed, the consumer should choose the same bundle.

There are two important points that come out of this:

1. This is an expression of the belief that changes in behavior should come from changes in the set of available alternatives. Since the rescaling of prices and income do not affect the budget set, they should not affect the consumer's choice.

2. The second point is that nominal prices are meaningless in consumer theory. If you tell me that a loaf of bread costs \$10, I need to know what other goods cost before I can interpret the first statement. And, in terms of analysis, this means that we can always "normalize" prices by arbitrarily setting one of them to whatever we like (often it is easiest to set it equal to 1), since only the real prices matter and fixing one commodity's nominal price will not affect the relative values of the other prices.

Exercise 3 *If you don't believe me that this change doesn't affect the budget set, you should go back to the two-commodity example, plug in the numbers and check it for yourself. If you can't do it with the general scaling factor* α*, you should let* $\alpha = 2$ *and try it for that. Most of the time, things that are hard to understand with general parameter values like* α, p, w *are simple once you plug in actual numbers for them and churn through the algebra.*

2.6.1 Comparative Statics of Homogeneity of Degree Zero

We can also perform a comparative statics analysis of the requirement that demand be homogeneous of degree zero, i.e. only real opportunities matter. What does this imply for choice behavior?

The homogeneity assumption applies to proportional changes in all prices and wealth:

$$x_i(\alpha p, \alpha w) \equiv x_i(p, w) \text{ for all } i,\ \alpha > 0.$$

To make things clear, let $s = (s_1, ..., s_L)$ denote the original price vector and v denote original wealth, and (for the time being) assume that $L = 2$. For example, (s, v) could be $s = (3, 2)$ and $v = 7$. Before we differentiate, I want to make sure that we're clear on what is going on. So, rewrite the above expression as:

$$x_i(\alpha s_1, \alpha s_2, \alpha v) \equiv x_i(s_1, s_2, v). \tag{2.5}$$

Now, notice that on the left-hand side, the first argument of x_i (which we usually call the "p_1 argument") is evaluated at the point $p_1 = \alpha s_1$, the second argument is evaluated at $p_2 = \alpha s_2$, and the third argument is evaluated at $w = \alpha v$.

We are interested in what happens to demand as α changes, so let's emphasize that what we care about is α by writing $g_1(\alpha) = \alpha s_1$, $g_2(\alpha) = \alpha s_2$, and $g_w(\alpha) = \alpha v$. Expression (2.5) can be rewritten as:

$$x_i\left(g_1(\alpha), g_2(\alpha), g_w(\alpha)\right) \equiv x_i\left(s_1, s_2, v\right) \tag{2.6}$$

This should make it clear that the p_1 argument in (2.5) and (2.6) contains a function of α, namely $p_1 = g_1(\alpha)$. Because of this, we have to use the Chain Rule in evaluating the derivative with respect to α. Differentiating (2.6) with respect to α yields:

$$\begin{aligned}\frac{\partial x_i\left(g_1(\alpha), g_2(\alpha), g_w(\alpha)\right)}{\partial p_1} g_1^{'}(\alpha) + \frac{\partial x_i\left(g_1(\alpha), g_2(\alpha), g_w(\alpha)\right)}{\partial p_2} g_2^{'}(\alpha) & \\ + \frac{\partial x_i\left(g_1(\alpha), g_2(\alpha), g_w(\alpha)\right)}{\partial w} g_w^{'}(\alpha) & \equiv 0,\end{aligned}$$

or

$$\frac{\partial x_i\left(\alpha s_1, \alpha s_2, \alpha v\right)}{\partial p_1} s_1 + \frac{\partial x_i\left(\alpha s_1, \alpha s_2, \alpha v\right)}{\partial p_2} s_2 + \frac{\partial x_i\left(\alpha s_1, \alpha s_2, \alpha v\right)}{\partial w} v \equiv 0. \tag{2.7}$$

Notice that the first line takes the standard Chain Rule form: for each argument (p_1, p_2, and w), take the partial derivative of the function with respect to that argument and multiply it by the derivative with respect to α of "what's inside" the argument.[12]

Finally, notice that (2.7) has prices and wealth $(\alpha s_1, \alpha s_2, \alpha v)$. Implicitly, when we are differentiating, we are asking the question "what happens to x_i when prices and wealth begin at (s_1, s_2, v) and are all increased slightly by the same proportion?" In order to make sure we are answering this question, we need to set $\alpha = 1$. Evaluating the last expression at $\alpha = 1$ yields the following expression in terms of the original price-wealth vector (s_1, s_2, v) :

$$\frac{\partial x_i\left(s_1, s_2, v\right)}{\partial p_1} s_1 + \frac{\partial x_i\left(s_1, s_2, v\right)}{\partial p_2} s_2 + \frac{\partial x_i\left(s_1, s_2, v\right)}{\partial w} v \equiv 0. \tag{2.8}$$

Generalizing the previous argument to the case where L is any positive number, expression (2.8) becomes:

$$\frac{\partial x_i(s, v)}{\partial w} v + \sum_{j=1}^{L} \frac{\partial x_i(s, v)}{\partial p_j} s_j = 0 \text{ for all } i. \tag{2.9}$$

This is where we need to face an ugly fact. Economists are terrible about notation, which makes this stuff harder to learn than it needs to be. When you see (2.9) written in a textbook, it will look like this:

$$\frac{\partial x_i(p, w)}{\partial w} w + \sum_{j=1}^{L} \frac{\partial x_i(p, w)}{\partial p_j} p_j = 0 \text{ for all } i.$$

But, notice that the symbol "w" in this expression has two different meanings. The "w" in "∂w" in the denominator of the first term says "we're differentiating with respect to the wealth argument,"

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[12] If you are confused, see the next subsection for further explanation.

while the "w" in "$\partial x_i(p,w)$" and the "w" multiplying this term refer to the original wealth level, i.e., the wealth level at which the expression is being evaluated. Similarly, "p_j" also has two different meanings in this expression. To make things worse, economists frequently skip steps in derivations.[13]

It is straightforward to get an elasticity version of (2.9). Just divide through by $x_i(s,v)$:

$$\varepsilon_{iw} + \sum_{j=1}^{L} \varepsilon_{ip_j} = 0. \tag{2.10}$$

Elasticities ε_{iw} and ε_{ip_j} give the elasticity of the consumer's demand response to changes in wealth and the price of good j, respectively. The total percentage change in consumption of good i is given by summing the percentage changes due to changes in wealth and in each of the prices. Homogeneity of degree zero says that in response to proportional changes in all prices and wealth the total change in demand for each commodity should not change. This is exactly what (2.10) says.

2.6.2 A Mathematical Aside ...

If this is unfamiliar to you, the computation may seem strange. If it doesn't seem strange, then skip on to the next section.

If you're still here, let's try it one more time. This time, we'll let $L = 2$, and choose specific values for the prices and wealth. Let good 1's price be 5, good 2's price be 3, and wealth be 10 initially. Then, (2.5) writes as:

$$x_i(5\alpha, 3\alpha, 10\alpha) \equiv x_i(5,3,10).$$

Now, starting at prices $(5,3)$ and wealth 10, we are interested in what happens to demand for x_i as we increase all prices and wealth proportionately. To do this, we will first increase α by a small amount (i.e., differentiate with respect to α), and then we'll evaluate the resulting expression at $\alpha = 1$. This will give us an expression for the effect of a small increase in α. So, totally differentiate both sides with respect to α :

$$\begin{aligned}
\frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_1}\frac{d(5\alpha)}{d\alpha} + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_2}\frac{d(3\alpha)}{d\alpha} + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial w}\frac{d(10\alpha)}{d\alpha} &\equiv 0 \\
\frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_1}5 + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_2}3 + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial w}10 &\equiv 0.
\end{aligned}$$

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[13] These are a couple of the main reasons why documents such as these are needed.

Again, the partial derivatives $\frac{\partial x_i}{\partial p_j}$ denote the partial derivative of function $x_i(p, w)$ with respect to the "p_j slot," i.e., the j^{th} argument of the function. And, since we are interested in what happens when you increase all prices and wealth proportionately beginning from prices $(5, 3)$ and wealth 10, we would like the left-hand side to be evaluated at $(5, 3, 10)$. To get this, set $\alpha = 1$:

$$\frac{\partial x_i(5,3,10)}{\partial p_1}5 + \frac{\partial x_i(5,3,10)}{\partial p_2}3 + \frac{\partial x_i(5,3,10)}{\partial w}10 \equiv 0 \tag{2.11}$$

Comparing this expression with (2.8) shows that the role of p_1^*, p_2^*, and w^* are played by 5, 3, and 10, respectively in (2.11), as is expected.

The source of confusion in understanding this derivation seems to lie in confusing the partial derivative of x_i with respect to the p_1 **argument** (for example) with the particular price of good 1, which is 5α in this example and αp_1^* in the more general derivation above. The key is to notice that, in applying the chain rule, you always differentiate the function (e.g., $x_i()$) with respect to its argument (e.g., p_1), and then differentiate the function that is in the argument's "slot" (e.g., 5α or αs_1 or αp_1 if you are an economist) with respect to α.

2.7 Requirement 3 Revisited: The Weak Axiom of Revealed Preference

The third requirement that we will place on consumer choices is that they satisfy the Weak Axiom of Revealed Preference (WARP). To remind you of the informal definition, WARP is a requirement of consistency in decision-making. It says that if a consumer chooses z when y was also affordable, this choice reveals that the consumer prefers z to y. Since we assume that consumer preferences are constant and we have modeled all of the relevant constraints on consumer behavior and preferences, if we ever observe the consumer choose y, it must be that z was not available (since if it were, the consumer would have chosen z over y since she had previously revealed her preference for z). We now turn to the formal definition.

Definition 4 *Consider any two distinct price-wealth vectors* (p, w) *and* $(p', w') \neq (p, w)$. *Let* $z = x(p, w)$ *and* $y = x(p', w')$. *The consumer's demand function satisfies WARP if whenever* $p \cdot y \leq w$, $p' \cdot z > w'$.

We can restate the last part of the definition as: if $y \in B_{p,w}$, then $z \notin B_{p',w'}$. If y could have been chosen when z was chosen, then the consumer has revealed that she prefers z to y. Therefore