Data: $x_1^0 = 100$, $p_1^0 = 10$, $\frac{\partial x}{\partial p} = -4$, $\frac{\partial x}{\partial w} = 0.02$. Note: there is no information on w.

Suppose the price of good 1 increases to $p' = 12.5$. How much should a public assistance program aimed at maintaining a certain standard of living be increased to offset this price increase?

To answer this question, we are looking for the CV of the price change. To compute this, we need to approximate the Hicksian demand curve for the original utility level, $h_1\left(p_1, p_{-1}, u^0\right)$.

1. We know that $h_1\left(10, p_{-1}, u^0\right) = x_1\left(10, p_{-1}, w\right)$.

2. The slope of the $h_1\left(p_1, p_{-1}, u^0\right)$ can be approximated using the data and the Slutsky equation.

$$\begin{aligned}\frac{\partial h}{\partial p} &= \frac{\partial x}{\partial p} + \frac{\partial x}{\partial w} x \\ &= -4 + 0.02\,(100) \\ &= -2\end{aligned}$$

3. So, at price 12.5, Hicksian demand is given by

$$\begin{aligned}h &= 100 + \frac{\partial h}{\partial p} dp \\ &= 100 + (-2)\,2.5 = 95\end{aligned}$$

4. To compute CV, compute the area of a trapezoid (or the area of a rectangle plus a triangle):

$$|CV| = (2.5)\left(\frac{95 + 100}{2}\right) = 243.75.$$

Since the price is increasing, we know that $CV < 0$, so $CV = -243.75$.

We could also estimate the change in Marshallian Consumer Surplus. This is just the area to the left of the Walrasian demand curve between the two prices. Hence $\Delta CS = -\,(2.5)\left(\frac{90+100}{2}\right) = -237.5$. Hence if we were to use the Marshallian consumer surplus in this case, we would not compensate the consumer enough for the price increase.

Another thing we could do is figure that the harm done to the consumer is just the change in price times the original consumption of this good, i.e., $2.5\,(100) = 250$. However, if we gave the consumer 250 additional dollars, we would be overcompensating for the price increase.

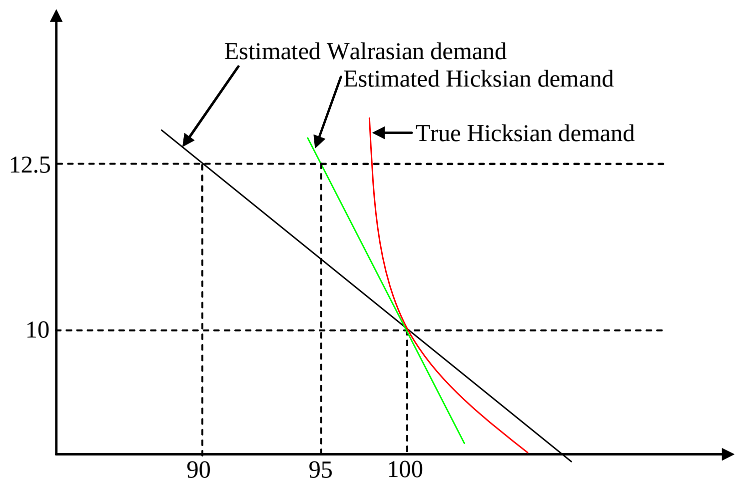
Estimated Walrasian demand
Estimated Hicksian demand
True Hicksian demand
12.5
10
90
95
100

Figure 1: